ATOMIC MOLECULAR (SAMPLE ASSIGNMENT)

Assignment Pedia
Complexity Simplified

1 Average lifetime of excited carriers.

The photoluminescence spectrum is plotted at two different time delays after $2 \times 10^{24}\ m^{-3}$ carriers have been excited using an ultra-short laser pulse.

a) Why is the photoluminescence spectrum time-dependent? What is the underlying physical phenomemon?

b) If the effective carrier temperature was 0 K, calculate what the electron Fermi energy would be for the initial carrier density. Take $m_e^* = 0.039m_0$.

c) Do the same for the hole Fermi energy. Assume that the densities of states from the light and heavy hole bands can just be added together. Take $m_{hh}^* = 0.45m_0$ and $m_{lh}^* = 0.05m_0$.

d) The effective carrier temperature for the 24 ps spectrum is 180 K. Are the carriers degenerate? Explain why.

e) Propose a graphical method to find the electron Fermi energy 24 ps after the excitation. Is the value that you find consistent with the answer you gave in b)?

f) Using the same idea as in e), find an estimate of the carrier density at 250 ps. Hence estimate the average lifetime of the carriers.

1 Average lifetime of excited carriers.

a) As time goes by, electrons excited in the conduction band and holes excited in the valence band relax to lower energy states, causing the photoluminescence spectrum to shift down to smaller energies.

b) When T = 0, the occupation factor is equal to 1 up to the Fermi level, and sharply drops to 0 above the Fermi level. In this case, the density of excited carriers is simply linked to the Fermi level through the formula (p. 101),

$$E_F^c = \frac{\hbar^2}{2m_e^*}(3\pi^2 N_e)^{\frac{2}{3}}$$

So, by substituting the numerical values,

$$E_F^c = \frac{(1.05 \times 10^{-34})^2}{2 \times 0.039 \times 9.11 \times 10^{-31}} \times (3\pi^2 \times 2 \times 10^{24})^{\frac{2}{3}}$$

$$E_F^c = 2.36 \times 10^{-20} J = 0.145eV$$

c) We apply the same equation for holes, but have to be a little careful dealing with the two bands. The total number of holes, N_{tot}, is the sum of N_{hh} and N_{lh}, respectively the numbers of heavy and light holes. As a result of Equation (1) applied to each kind, we have,

$$3\pi^2 N_{hh} = \left(\frac{2m_{hh}E_F^v}{h^2}\right)^{\frac{3}{2}}$$

$$3\pi^2 N_{lh} = \left(\frac{2m_{lh}E_F^v}{h^2}\right)^{\frac{3}{2}}$$

Adding those two lines,

$$3\pi^2 N_{tot} = \left(\frac{2E_F^v}{h^2}\right)^{\frac{3}{2}}\left(m_{hh}^{\frac{3}{2}} + m_{lh}^{\frac{3}{2}}\right)$$

Therefore, we get the hole Fermi energy as,

$$E_F^v = \frac{\hbar^2}{2\left(m_{hh}^{\frac{3}{2}} + m_{lh}^{\frac{3}{2}}\right)^{\frac{2}{3}}} (3\pi^2 N_{tot})^{\frac{2}{3}}$$

$$E_F^v = 0.0125eV$$

Note: The analytical expression of the reduced mass μ is a result of the derivation. Here we find $\mu = \left(m_{hh}^{\frac{3}{2}} + m_{lh}^{\frac{3}{2}}\right)^{\frac{2}{3}}$, which is different from the usual $1/\mu = 1/m_1 + 1/m_2$.

d) In order to know whether the carriers are degenerate, we have to compare $k_B T$ with the Fermi level. For T = 180 K, $k_B T = 0.015$ eV. This is of the order of E_F^v and much smaller than E_F^c. So, the electrons are degenerate, but not the holes.

e) $E_g + E_F^c$ may be read from the spectrum at about 0.95 eV as the point where the luminescence falls to 50% of its peak value. Since $E_g \sim 0.81$ eV, this yields $E_F^c = 0.14$ eV, which is close to what we found in question b).

f) Using the same graphical method at 250 ps, we find $E_F^c = 0.04$ eV. Then,

$$N_{250} = \frac{1}{3\pi^2}\left(\frac{2m_e E_F^c}{\hbar^2}\right)^{\frac{3}{2}} = 2.83 \times 10^{23} m^{\ \ 3}$$

The number of excited carriers follows an exponential decay law,

$$N(t) = N_0 e^{-\frac{t}{\tau}}$$

Therefore,

$$\tau = \frac{t}{ln(N_0/N_{250})} = \frac{250ps}{ln\left(\frac{2\times10^{24}}{2.83\times10^{23}}\right)} = 0.13ns$$

2 Laser oscillation.

a) Explain briefly how a laser differs from a simple light emitting diode.

Consider a laser made up of a 10 cm long ruby rod (refractive index = 1.76).

b) Calculate the reflectivity of the ruby-air interface.

c) Calculate the frequency separation of the longitudinal modes.

d) Suppose the ruby rod is coated so that one end has a reflectivity of 95%. The other end is uncoated. Calculate the threshold gain coefficient for the laser if the scattering and other impurity losses are negligibly small.

2 Laser oscillation.

a) A simple light emitting diode only uses spontaneous luminescence, when the laser effect is based on *stimulated emission*.

b) The formula for the reflectivity is,

$$R = \left[\frac{n-1}{n+1}\right]^2$$

where n is the refractive index. So here,

$$R = \left[\frac{1.76 \quad 1}{1.76+1}\right]^2 = 7.6\%$$

c) The frequency separation of the longitudinal modes is,

$$\frac{c}{2nl} = \frac{3 \times 10^8}{2 \times 1.76 \times 0.1} = 8.5 \times 10^8 Hz$$

d) In the absence of other losses than the ones due to transmission at the ends of the rod, we simply have,

$$\gamma_{th} = -\frac{1}{2l} ln(R_1 R_2) = -\frac{1}{2 \times 0.1} ln(0.076 \times 0.95) = 13.14 m^{-1}$$

3 Power efficiency.

A laser diode emits at 800 nm when operating at an injection current of 90 mA.

a) Calculate the maximum possible power than can be emitted by the device.

b) Calculate the power conversion efficiency, if the actual power output is 45 mW and the operating voltage is 1.8 V.

c) The threshold current of the laser is 30 mA. What is the slope efficiency and the quantum efficiency?

3 Power efficiency.

a) If the quantum efficiency is 100% and there are no losses of any kind, each incoming electron causes an excitation which in turn leads to the stimulated emission of one photon. The number of photons emitted per second is then $\frac{i}{e} = \frac{90 \times 10^{\;3}}{1.6 \times 10^{-19}} = 5.63 \times 10^{17}\ s^{-1}$, and each photon carries an energy equal to $\frac{c}{\lambda} h = \frac{3 \times 10^8}{800 \times 10^{\;9}} \times 6.63 \times 10^{-34} = 2.49 \times 10^{-19}\ J$. Therefore, the output power is $5.63 \times 10^{17} \times 2.49 \times 10^{-19} = 140\ mW$.

b) $\frac{45}{1.8 \times 90} = 27.8\%$.

c) Now introducing a threshold current,

$$P_{out} = \eta \frac{h\nu}{e} (I_{in} - I_{th})$$

$$\eta = \frac{P_{out} \frac{e}{h\nu}}{I_{in} - I_{th}} = 48\%$$